

between theory and application ought not yet to be insurmountable.

“Of course, we recognize that there are obstacles which cannot be made to disappear. One of them is the language itself; we have kept the mathematical notations to a minimum, and indexed them (with definitions) at the end of the book. We also know that, even after a norm has been interpreted as a natural measure of strain energy, and a Hilbert space identified with the class of admissible functions in a physically derived variational principle, there still remains the hardest problem: to become comfortable with these ideas, and to make them one's own. This requires genuine patience and tolerance on both sides, as well as effort. Perhaps this book at least exhibits the kind of problems which a mathematician is trained to solve, and those for which he is useless.”

E. I.

36 [5, 6]. — I. I. GIHMAN & A. V. SKOROHOD, *Stochastic Differential Equations*, translated from the Russian, Springer Verlag, New York, 1972, viii + 354 pp., 24 cm. Price \$27.90.

This book is a treatise on stochastic equations by two well-known authors in probability theory. It is a theoretical work and much attention is devoted to developing the foundations: existence, uniqueness, regularity, etc. The book presents many other results; for example it treats asymptotic behavior, which should be of direct interest to readers with applied interests. The authors do not treat specific applied problems in detail. Let us give a brief review of the contents of the book which is arranged in two parts.

Part I deals with one-dimensional stochastic differential equations of first order exclusively. Chapter 1 gives a succinct treatment of K. Itô's theory of stochastic integrals. Chapter 2 deals with existence and uniqueness, again following K. Itô's methods, i.e., a stochastic Picard iteration method. The authors are careful here, as well as in the rest of the book, to single out the necessary hypotheses and avoid unnatural restrictions. In Chapter 2, they give a thorough analysis of the dependence of solutions on parameters. This is important in establishing the connection of stochastic differential equations and partial differential equations. Chapter 3 analyzes the connection just mentioned and the Markov character of the solutions of stochastic equations. One finds that probabilistic methods yield very strong results on the existence, uniqueness, and regularity of parabolic partial differential equations. Moreover, these results do not depend upon uniform ellipticity assumptions. Chapter 4 deals with the asymptotic behavior of the solutions of stochastic equations. The results of this chapter are very sharp and many are not available elsewhere. Chapter 5 treats problems on a finite interval and gives a thorough analysis of boundary behavior as well as asymptotic behavior.

Part II examines systems of stochastic differential equations. Chapter 1

deals with vector stochastic equations in general and not only with stochastic equations that lead to Markov processes. Many of the concepts here are due to the authors. Chapter 2 specializes in stochastic equations without after-effect (i.e., Markov) but includes jump processes as well as diffusions. The existence, uniqueness, and regularity theory is not given in detail again since it is similar to that of Part I. The results are stated in full, however. The latter sections of Chapter 2 contain a wealth of information on the connections between functionals of solutions of stochastic equations and partial differential equations. Chapter 3 is perhaps the most important chapter in the book from the point of view of applications. A number of very strong results on asymptotic behavior for large time and as a parameter tends to a limiting value are obtained. Bogoliubov's averaging method is extended to stochastic equations.

This book, along with H. P. McKean's "Stochastic Integrals", Academic Press, New York, 1968, provide excellent foundations and up to date information on stochastic differential equations.

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37[7].—H. P. ROBINSON, *Tables of the Derivative of the Psi Function to 58 Decimals*, University of California, Lawrence Berkeley Laboratory, Berkeley, California, October 1973. Ms. of 11 pp. deposited in the UMT file.

This unpublished set of tables consists of 58D values of the trigamma function, $\Psi'(x)$, for $x = n + a$, where $n = 0(1)50$ and $a = 0, 1/2, 1/3, 2/3, 1/4, 3/4, 1/5, 2/5, 3/5, 4/5$.

The tabular values were calculated on a Wang 720C electronic calculator by means of the (stable) backward recursion formula $\Psi'(x-1) = \Psi'(x) + (x-1)^{-2}$, starting with values corresponding to $x = 1000 + a$, which were calculated by the appropriate asymptotic series. The terminal values in this recurrence were checked by the reflection formula $\Psi'(x) + \Psi'(1-x) = \pi^2 \csc^2 x$.

It may be appropriate to remark here that these excellent tables possess much higher precision than published tables [1], [2] of this function, which extend to at most 19D.

J. W. W.

1. BRITISH ASSOCIATION FOR THE ADVANCEMENT OF SCIENCE, *Mathematical Tables*, v. 1, third edition, Cambridge University Press, Cambridge, England, 1951.

2. H. T. DAVIS, *Tables of the Mathematical Functions*, second edition, v. 2, The Principia Press of Trinity University, San Antonio, Texas, 1963. (For references to additional tables, with lower precision, see A. FLETCHER, J. C. P. MILLER, L. ROSENHEAD, AND L. J. COMRIE, *An Index of Mathematical Tables*, second edition, v. 1, Addison-Wesley, Reading, Massachusetts, 1962, p. 298.)